$$\frac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$$

$$\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}} = \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$$

$$\frac{\cos\theta+1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta+\sin\theta\cos\theta} =$$

$$\frac{\cancel{\cos\theta+1}}{\sin\theta(\cancel{1+\cos\theta})} = \frac{1}{\sin\theta} = \csc\theta$$

(4a) $$\frac{2-\sec^2\theta}{\sec\theta} = \frac{1-2\sin^2\theta}{\cos\theta}$$

$$\frac{2}{\sec\theta} - \frac{\sec^2\theta}{\sec\theta}$$

$$2\cos\theta - \sec\theta$$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{2\cos\theta}{1} - \frac{1}{\cos\theta}$$

$$\frac{2\cos^2\theta-1}{\cos\theta} = \frac{2(1-\sin^2\theta)-1}{\cos\theta} =$$

$$\frac{2-2\sin^2\theta-1}{\cos\theta} = \frac{1-2\sin^2\theta}{\cos\theta}$$

$$\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$$

$$\frac{1}{(\sec\theta - \tan\theta)} \cdot \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$\frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$\frac{\sec\theta + \tan\theta}{1}$$

$$\sec\theta + \tan\theta$$

$$\frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \frac{1}{\frac{1-\sin\theta}{\cos\theta}}$$

$$= 1 \cdot \frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta} \frac{(1+\sin\theta)}{(1+\sin\theta)}$$

$$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} = \frac{\cos\theta(1+\sin}{\cos^2\theta}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

$\cot\theta(\sin\theta + \tan\theta) = 1 + \cos\theta$

$\cot\theta\sin\theta + \cot\theta\tan\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} + \frac{1}{\cancel{\tan\theta}} \cdot \cancel{\tan\theta}$

$\cos\theta + 1$

45 $\tan\theta(\cot\theta - \cos\theta) = 1 - \sin\theta$

$\tan\theta\cot\theta - \tan\theta\cos\theta$

$\cancel{\tan\theta} \cdot \frac{1}{\cancel{\tan\theta}} - \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

$1 - \sin\theta$

$$\frac{\tan\theta}{1+\tan^2\theta} = \sin\theta\cos\theta$$

$$\frac{\tan\theta}{\sec^2\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$

$$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{1}$$

$$\sin\theta\cos\theta$$

47 $$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

$$\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$\frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$\frac{(1-\sin\theta)^{\cancel{2}}}{(1+\sin\theta)\cancel{(1-\sin\theta)}} = \frac{1-\sin\theta}{1+\sin\theta}$$

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta}$$

$$\frac{\cos\theta)\cdot(1-\cos\theta)}{\cos\theta)\,(1-\cos\theta)}$$

$$\frac{-\cos\theta)^2}{-\cos^2\theta}$$

$$\frac{-\cos\theta)^2}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$$

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$$\sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \sqrt{\frac{(\sec\theta+\tan\theta)(\sec\theta+\tan\theta)}{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}}$$

$$= \sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}} = \sqrt{\frac{(\sec\theta+\tan\theta)^2}{1}}$$

$$= \sec\theta+\tan\theta$$

$$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

50) $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

$(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$1(\sin^2\theta - \cos^2\theta)$

$1 - \cos^2\theta - \cos^2\theta$

$1 - 2\cos^2\theta$

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